Published by Institute of Physics Publishing for SISSA/ISAS
Received: *July 29, 2003*
Accepted: *August 29, 2003*

D-branes in $N = 2$ strings

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Abstract: We study various aspects of D-branes in the two families of closed $N = 2$ strings denoted by α and β in [3]. We consider two types of $N = 2$ boundary conditions, A-type and B-type. We analyse the D-branes geometry. We compute open and closed string scattering amplitudes in the presence of the D-branes and discuss the results. We find that, except the space filling D-branes, the B-type D-branes decouple from the bulk. The A-type D-branes exhibit inconsistency. We construct the D-branes effective worldvolume theories. They are given by a dimensional reduction of self-dual Yang-Mills theory in four dimensions. We construct the D-branes gravity backgrounds. Finally, we discuss possible $N = 2$ open/closed string dualities.

Keywords: Superstrings and Heterotic Strings, D-branes.

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1. Introduction

Closed $N = 2$ strings [1, 2] possess local $N = 2$ supersymmetry on the string worldsheet. Critical $N = 2$ strings have a four-dimensional target space. The supersymmetric structure implies that the target space has a complex structure. Therefore it must be of signature $(4, 0)$ or $(2, 2)$. In $(4, 0)$ signature there are no propagating degrees of freedom in the $N = 2$ string spectrum. In $(2, 2)$ signature there are only massless scalars in the spectrum and the infinite tower of massive excitations of the string is absent.

In [3] the $N = 2$ closed strings have been divided into two families denoted by α and β. Consider $N = 2$ strings in a flat background. In order to construct the $N = 2$ string we need to gauge the $N = 2$ superconformal algebra (SCA) on the worldsheet. More precisely we have two copies of the $N = 2$ algebra to consider: the left and right sectors. The free field representation of the (left) $N=2$ SCA takes the form

$$\begin{aligned}
T &= -\frac{1}{2}\eta_{IJ}\partial x^I\partial x^J - \frac{1}{4}\eta_{IJ}\left(\partial\psi^I\psi^J + \partial\psi^J\psi^I\right) , \\
J &= \frac{i}{2}\mathcal{J}^L_{IJ}\psi^I\psi^J , \\
G^{\pm} &= \frac{i}{2}\left(\eta_{IJ} \pm i\mathcal{J}^L_{IJ}\right)\psi^I\partial x^J . \qquad (1.1)
\end{aligned}$$

Here $I, J = 1, \ldots, 4$ denote the indices of the target space in a real basis. The metric is given by $\eta_{IJ} = \mathrm{diag}(-1,-1,+1,+1)$. $\mathcal{J}^L_{IJ}$ is a Kähler form related to the complex structure $\mathcal{J}^K_J$ by $\mathcal{J}_{IJ} = \eta_{IK}\mathcal{J}^K_J$, and the index L refers to the left sector. Similarly, we have the SCA generators in the right sector with a complex structure $\mathcal{J}^R$. The $N=2$ string denoted by β-string in [3] is defined by having the same complex structure in the left and right sectors $\mathcal{J}^L = \mathcal{J}^R$. The $N=2$ string denoted as α-string in [3] has different (inequivalent) complex structures in the left and right sectors.

The β- and α-strings define two families of $N=2$ strings related by T-duality [3]. In $(2,2)$ signature both families have one scalar in the spectrum. The effective action of the β-string scalar has been computed in [4], which suggested its interpretation as a deformation of the target space Kähler potential. The effective action of the scalar in the α-string has been computed in [5]. It was found that the α-string scalar is free and that the dynamics is that of a self-dual curvature with torsion considered in [6].

In this paper we study the D-branes in both families of $N=2$ strings. We will mainly consider the target space with $(2,2)$ signature. The extension to $(4,0)$ signature is straightforward. We will consider two types of $N=2$ boundary conditions, A-type and B-type [7, 3]. We will compute open and closed string scattering amplitudes in the presence of the D-branes and discuss the results. We find that, except the space filling D-branes, the B-type D-branes decouple from the bulk: the perturbative closed string scattering amplitudes off B-type branes vanish. The A-type D-branes exhibit an inconsistency: the scattering amplitude of two closed string modes in the A-branes background contains an infinite number of poles that correspond to massive excitations of both open and closed string states. Such an observation was first made in [8].

We construct the D-branes effective worldvolume theories. They are given by a dimensional reduction of self-dual Yang-Mills theory in four dimensions. They correspond to various integrable systems: Bogomolny equation [9] for the three-branes, the Hitchin system [10] for the two-branes, Nahm equations for the one-branes [11] and the ADHM equation [12] for the zero-branes. We construct the D-branes gravity backgrounds. Finally, we comment on $N=2$ open/closed string dualities.

The paper is organized as follows. In section 2 we consider the A-type and B-type boundary conditions on the $N=2$ algebra. We discuss the classification of $N=2$ D-branes and their geometry. In section 3 we construct vertex operators and compute open and closed string scattering amplitudes in the presence of the D-branes. In section 4 we construct the D-branes effective worldvolume theories as a dimensional reduction of

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self-dual Yang-Mills theory in four dimensions and see the correspondence to integrable systems. In section 5 we construct the D-branes gravity backgrounds. In section 6 we discuss possible $N = 2$ open/closed string dualities.

2. $N = 2$ boundary conditions

In this section we consider the A-type and B-type boundary conditions on the $N = 2$ algebra and the corresponding $N = 2$ D-branes. We will expand the discussion of [3, section 7].

2.1 Closed $N = 2$ strings

In the following we will review some aspects of the complex structures on $\mathbb{R}^{2,2}$ that are relevant to the generators of $N = 2$ SCA. In the real basis $x^I = (x^1, x^2, x^3, x^4)$, the metric is given by $\eta_{IJ} = \mathrm{diag}(-1, -1, +1, +1)$. We define a complex structure

$$\mathcal{J}^I{}_J = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix} . \tag{2.1}$$

In the complex basis

$$z^1 = \frac{x^1 + ix^2}{\sqrt{2}} , \qquad z^2 = \frac{x^3 + ix^4}{\sqrt{2}} , \tag{2.2}$$

the metric reads $\eta_{i\bar{j}} = \mathrm{diag}(-1, +1)$, $i, \bar{j} = 1, 2$. In this basis, the complex structure $\mathcal{J}^I{}_J$ is diagonal:

$$\begin{aligned} \mathcal{J}(z^1) &= -iz^1 , \qquad & \mathcal{J}(\bar{z}^{\bar{1}}) &= +i\bar{z}^{\bar{1}} , \\ \mathcal{J}(z^2) &= -iz^2 , \qquad & \mathcal{J}(\bar{z}^{\bar{2}}) &= +i\bar{z}^{\bar{2}} . \end{aligned} \tag{2.3}$$

The Kähler form $\mathcal{J}_{IJ} = \eta_{IK}\mathcal{J}^K_J$ is given in the real basis by

$$\mathcal{J}_{IJ} = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix} . \tag{2.4}$$

For later reference, we define the quadratic form in momenta

$$k_i^I \mathcal{J}_{IJ} k_j^J = ic_{ij} , \tag{2.5}$$

where in the complex basis

$$c_{ij} = k_i \cdot \bar{k}_j - \bar{k}_i \cdot k_j , \tag{2.6}$$

and $k_i \cdot \bar{k}_j \equiv \eta_{m\bar{n}} k_i^m \bar{k}_j^{\bar{n}}$. For the on-shell momenta k_i, $i = 1, 2, 3, 4$ with $k_i^2 = 0$, $k_1 + k_2 + k_3 + k_4 = 0$, c_{ij} obey the identities [4]

$$\frac{c_{12}c_{34}}{s} + \frac{c_{23}c_{41}}{t} = u , \qquad \frac{c_{21}c_{34}}{s} + \frac{c_{13}c_{42}}{u} = t , \qquad \frac{c_{13}c_{24}}{u} + \frac{c_{32}c_{41}}{t} = s , \tag{2.7}$$

where $s = -k_1 \cdot k_2 \equiv -(k_1 \cdot \bar{k}_2 + \bar{k}_1 \cdot k_2)$, $t = -k_2 \cdot k_3$, $u = -k_1 \cdot k_3$.

We will also need a second complex (and Kähler) structure, which in the real basis take the form

$$\widetilde{\mathcal{J}}^I{}_J = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix} , \qquad \widetilde{\mathcal{J}}_{IJ} = \eta_{IK}\widetilde{\mathcal{J}}^K_J = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix} . \tag{2.8}$$

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In the complex basis, the complex structure is given by

$$\begin{aligned} \widetilde{\mathcal{J}}(z^1) &= -iz^1\,, \qquad & \widetilde{\mathcal{J}}(\bar{z}^{\bar{1}}) &= +i\bar{z}^{\bar{1}}\,, \\ \widetilde{\mathcal{J}}(z^2) &= +iz^2\,, \qquad & \widetilde{\mathcal{J}}(\bar{z}^{\bar{2}}) &= -i\bar{z}^{\bar{2}}\,. \end{aligned} \tag{2.9}$$

The Kähler form reads

$$\widetilde{\mathcal{J}}_{i\bar{j}} = -i\mathbf{1}\,, \qquad \widetilde{\mathcal{J}}_{\bar{i}j} = +i\mathbf{1}\,. \tag{2.10}$$

The β-string in $\mathbb{R}^{2,2}$ is defined by the choice

$$\mathcal{J}^L_{IJ} = \mathcal{J}^R_{IJ} = \mathcal{J}_{IJ}\,, \tag{2.11}$$

in (1.1), while for the α-string

$$\mathcal{J}^L_{IJ} = \mathcal{J}_{IJ}\,, \qquad \mathcal{J}^R_{IJ} = \widetilde{\mathcal{J}}_{IJ}\,. \tag{2.12}$$

The two $N=2$ strings are related by a T-duality along a spatial direction.

2.2 A-type and B-type boundary conditions

We consider the two types of $N=2$ boundary conditions: the A-type and B-type. In the closed string notation the boundary conditions read [7]:

- **A-type**:

$$\left(L_n - \widetilde{L}_{-n}\right)|\,\rangle\!\rangle_A = \left(J_n - \widetilde{J}_{-n}\right)|\,\rangle\!\rangle_A = \left(G^{\pm}_n - i\widetilde{G}^{\mp}_{-n}\right)|\,\rangle\!\rangle_A = 0\,, \tag{2.13}$$

- **B-type**:

$$\left(L_n - \widetilde{L}_{-n}\right)|\,\rangle\!\rangle_B = \left(J_n + \widetilde{J}_{-n}\right)|\,\rangle\!\rangle_B = \left(G^{\pm}_n - i\widetilde{G}^{\pm}_{-n}\right)|\,\rangle\!\rangle_B = 0\,. \tag{2.14}$$

In order to solve these equations we use the oscillator modes:

$$\left(\alpha^I_n - U^I{}_J\widetilde{\alpha}^J_{-n}\right)|\,\rangle\!\rangle_{A,B} = \left(\psi^I_n - iV^I{}_J\widetilde{\psi}^J_{-n}\right)|\,\rangle\!\rangle_{A,B} = 0\,. \tag{2.15}$$

The consistency condition that these operators (anti-)commute with each other implies that U and V are elements of $O(2,2)$. Using (2.15) in (2.13) and (2.14) and the free field representation of the $N=2$ algebra (1.1) we get

- **A-type**:

$$U = V\,, \qquad (U^T\mathcal{J}^L U)_{IJ} = -\mathcal{J}^R_{IJ}\,, \tag{2.16}$$

- **B-type**:

$$U = V\,, \qquad (U^T\mathcal{J}^L U)_{IJ} = +\mathcal{J}^R_{IJ}\,. \tag{2.17}$$

For the β-string $\mathcal{J}^L_{IJ} = \mathcal{J}^R_{IJ} = \mathcal{J}_{IJ}$ and we get

- **A-type**:

$$U = V = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \tag{2.18}$$

  defining a (1+1)-brane.

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- **B-type**:

$$U = V = \begin{pmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{pmatrix}, \tag{2.19}$$

defining a (2+2), (2+0), (0+2) and (0+0)-brane depending on the number of +1 eigenvalues.

For the α-string $\mathcal{J}^L_{IJ} = \mathcal{J}_{IJ}, \quad \mathcal{J}^R_{IJ} = \widetilde{\mathcal{J}}_{IJ}$ and we get

- **A-type**:

$$U = V = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \pm 1 \end{pmatrix}, \tag{2.20}$$

defining a (1+2) and (1+0)-brane.

- **B-type**:

$$U = V = \begin{pmatrix} \pm 1 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \tag{2.21}$$

defining a (2+1) and (0+1)-brane.

2.3 Geometry of D-branes

We start by considering $N = 2$ strings in a flat $\mathbb{R}^{2,2}$ background. The B-type boundary conditions in the β-string correspond to even-dimensional D-branes. The analysis of [7] implies that the worldvolumes are holomorphic (Kähler) submanifolds of $\mathbb{R}^{2,2}$ with the complex structure $\mathcal{J}$. The space-time filling (2+2)-brane is the target space $\mathbb{R}^{2,2}$ itself. If we use the complex coordinates on $\mathbb{R}^{2,2}$ z_1, z_2 in (2.2), then the (2+0) and (0+2)-branes are the holomorphic 2-cycles $[z_1, z_2 = 0]$ and $[z_1 = 0, z_2]$, respectively. The (0+0)-brane is a point in $\mathbb{R}^{2,2}$.

The A-type boundary conditions in the β-string correspond to a (1+1)-brane. The worldvolume cycle is a special lagrangian submanifold of $\mathbb{R}^{2,2}$ [7]. Denote the (1+1)-brane worldvolume coordinates by x^1, x^3. The pull-backs of the holomorphic 2-form Ω and Kähler 2-form $\mathcal{K}$ read

$$\Omega|_{\text{worldvolume}} = dx^1 \wedge dx^3\,, \qquad \mathcal{K}|_{\text{worldvolume}} = 0\,. \tag{2.22}$$

The A and B-type boundary conditions in the α-string correspond to odd-dimensional D-branes. We consider $\mathbb{R}^{2,2}$ as a product of two one-dimensional Kähler manifolds $\mathcal{M}_1$ and $\mathcal{M}_2$. Consider the (2+1)-brane. The worldvolume is a product of a two-dimensional cycle, say $M_1 = \mathcal{M}_1$ parametrized by x^1, x^2 and the one-dimensional cycle $M_2 \in \mathcal{M}_2$ parametrized by x^3. We will now show that M_1 is a holomorphic 2-cycle and M_2 is a lagrangian submanifold with respect to the complex structure $\mathcal{J}$ or $\widetilde{\mathcal{J}}$. We will follow the discussion in [13].

We write the matrix U (2.21) as

$$U = U^+ + U^-\,, \tag{2.23}$$

where

$$U^+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad U^- = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_3 \end{pmatrix}. \tag{2.24}$$

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U^+ defines the cycle M_1 and U^- defines the cycle M_2. It is easy to see that the following relations hold

$$U^+\mathcal{J} = +\mathcal{J}U^+\,, \qquad U^-\mathcal{J} = -\mathcal{J}U^-\,, \tag{2.25}$$

and

$$U^+\widetilde{\mathcal{J}} = +\widetilde{\mathcal{J}}U^+\,, \qquad U^-\widetilde{\mathcal{J}} = -\widetilde{\mathcal{J}}U^-\,. \tag{2.26}$$

These imply that M_1 is a holomorphic 2-cycle and M_2 is a lagrangian submanifold with respect to the complex structures $\mathcal{J}$ or $\widetilde{\mathcal{J}}$. The $(0+1)$-brane is a point in $\mathcal{M}_1$ times a lagrangian submanifold with respect to the complex structures $\mathcal{J}$ or $\widetilde{\mathcal{J}}$ in $\mathcal{M}_2$.

Consider next the D-branes in a curved background $\mathcal{M}$. The analysis of the D-branes in the β-string is done in [7] and the worldvolumes are holomorphic (sub)manifolds of $\mathcal{M}$. Consider next the α-string. The target space geometry of α-string is given by a bi-hermitean geometry with two commuting complex structures $\mathcal{J}^\pm$ that are covariantly constant with respect to the affine connection with a torsion $\nabla^\pm\mathcal{J}^\pm = 0$ with $\Gamma^{\pm\rho}{}_{\mu\nu} = \Gamma^\rho{}_{\mu\nu} \mp H^\rho_{\mu\nu}$ [14, 6]. One defines the projection operator [13]

$$\Pi_\pm = \frac{1}{2}\left(I \pm \Pi\right), \tag{2.27}$$

with

$$\Pi = \mathcal{J}^+\mathcal{J}^-\,, \qquad \Pi^2 = 1\,. \tag{2.28}$$

It defines a local product structure of the target space geometry: $\mathcal{M}$ is locally $\mathcal{M}_1 \times \mathcal{M}_2$ where $\mathcal{M}_1$ and $\mathcal{M}_2$ are Kähler manifolds. The D-branes have a local product structure as well. The matrix U can de decomposed as in (2.23) with

$$U^\pm = \Pi_\pm U \Pi_\pm\,. \tag{2.29}$$

The boundary conditions require the the solution of two independent relations in the Kähler geometry of $\mathcal{M}_1$ and $\mathcal{M}_2$

$$\begin{aligned} U^+\mathcal{J}_+ &= \eta\mathcal{J}_+U^+\,, \\ U^-\mathcal{J}_+ &= -\eta\mathcal{J}_+U^-\,, \end{aligned} \tag{2.30}$$

where $\eta = 1, -1$ for the B- and A- type boundary conditions, respectively. When $\eta = 1$ the brane geometry is a local product of a holomorphic cycle in $\mathcal{M}_1$ and a lagrangian cycle in $\mathcal{M}_2$ with respect to the complex structure $\mathcal{J}^+$. The structure is interchanged when $\eta = -1$.

3. $N = 2$ strings scattering off D-branes

In this section we compute string scattering amplitudes in the presence of A-type and B-type branes. We will see that in the presence of A-branes the amplitudes exhibit infinite number of massive poles, which do not correspond to $N = 2$ string states. The amplitudes in the presence of B-branes vanish and imply a decoupling between the open and closed string states. We note that $N = 2$ strings scatterings in the presence of D-branes were studied in [8], before their A and B classification [3]. There will be some overlap between our computations and those of [8].

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3.1 The vertex operators

We start by constructing the vertex operators of the closed $N=2$ string scalar ϕ in the different pictures. We consider both the α and β strings. We analyze first the matter part of the vertex operators. The ghost part will be discussed later.

The scalar vertex operator of the matter sector part reads in the $(-1,-1)$-picture

$$V_L^{(-1,-1)}(z) = e^{ik\cdot X_L(z)}\,, \qquad V_R^{(-1,-1)}(\bar z) = e^{ik\cdot X_R(\bar z)}\,. \tag{3.1}$$

The picture-changing operators [15] are given by the world sheet supercharges:

$$G_L^{\pm}(z) = \frac{i}{2}\left(\eta_{IJ} \pm i\mathcal{J}^L_{IJ}\right)\psi_L^I \partial X_L^J\,, \qquad G_R^{\pm}(\bar z) = \frac{i}{2}\left(\eta_{IJ} \pm i\mathcal{J}^R_{IJ}\right)\psi_R^I \bar\partial X_R^J\,. \tag{3.2}$$

Using the OPE's of the free fields

$$X_L^I(z)X_L^J(w) \sim -\eta^{IJ}\log(z-w)\,, \qquad \psi_L^I(z)\psi_L^J(w) \sim -\frac{\eta^{IJ}}{z-w}\,, \tag{3.3}$$

one gets

$$G_L^{\pm}(z)V_L^{(-1,-1)}(0) \sim \frac{1}{z}\left(V_L^{(0,-1)}(0),\, V_L^{(-1,0)}(0)\right)\,, \tag{3.4}$$

where

$$V_L^{(0,-1)}(z) = \frac{1}{2}(kJ_-^L\psi_L(z))\,e^{ik\cdot X_L(z)}\,, \qquad V_L^{(-1,0)}(z) = \frac{1}{2}(kJ_+^L\psi_L(z))\,e^{ik\cdot X_L(z)}\,. \tag{3.5}$$

Here we used $J_\pm^L \equiv \eta \pm i\mathcal{J}^L$. Also

$$G_R^{\pm}(\bar z)V_R^{(-1,-1)}(0) \sim \frac{1}{\bar z}\left(V_R^{(0,-1)}(0),\, V_R^{(-1,0)}(0)\right)\,, \tag{3.6}$$

where

$$V_R^{(0,-1)}(\bar z) = \frac{1}{2}(kJ_-^R\psi_R(\bar z))\,e^{ik\cdot X_R(\bar z)}\,, \qquad V_R^{(-1,0)}(\bar z) = \frac{1}{2}(kJ_+^R\psi_R(\bar z))\,e^{ik\cdot X_R(\bar z)}\,, \tag{3.7}$$

with $J_\pm^R \equiv \eta \pm i\mathcal{J}^R$. The vertex operator in the $(0,0)$-picture is

$$G_L^-(z)V_L^{(0,-1)}(0) - G_L^+(z)V_L^{(-1,0)}(0) \sim \frac{1}{z}\,V_L^{(0,0)}(0)\,, \tag{3.8}$$

and

$$V_L^{(0,0)}(z) = \left(-k\mathcal{J}^L\partial X_L + \frac{1}{2}\left(kJ_+^L\psi_L\right)\left(kJ_-^L\psi_L\right)\right)e^{ik\cdot X_L}\,. \tag{3.9}$$

The right sector vertex operators take a similar form with $L \to R$.

Consider next the open string vertex operators. We take the open strings on the upper-half plane with the real axis as a boundary. The vertex operator of the open $N=2$ scalar φ [16] reads in the $(-1,-1)$-picture

$$V_o^{(-1,-1)} = e^{ik\cdot X_L + ik\cdot X_R}\,, \tag{3.10}$$

with the boundary conditions for X and ψ given by

$$X_L^I(z) = D^I_J X_R^J(\bar z)\,, \qquad \psi_L^I(z) = D^I_J\psi_R^J(\bar z) \qquad \text{at} \quad \mathrm{Im} z = 0\,, \tag{3.11}$$

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where

$$D = \mathrm{diag}(\pm \pm \pm\pm)\,. \tag{3.12}$$

Here $+, -$ correspond to the Neumann (N), Dirichlet (D) boundary conditions, respectively.

It is useful to use the doubling technique to implement the boundary conditions. We define

$$\begin{aligned} x^I(z) &= \begin{cases} X_L^I(z)\,, & \mathrm{Im} z \geq 0 \\ D^I_J X_R^J(z)\,, & \mathrm{Im} z \leq 0 \end{cases} \\ \Psi^I(z) &= \begin{cases} \psi_L^I(z)\,, & \mathrm{Im} z \geq 0 \\ D^I_J \psi_R^J(z)\,, & \mathrm{Im} z \leq 0 \end{cases} \end{aligned} \tag{3.13}$$

with the OPE's given by

$$x^I(z)x^J(w) \sim -\eta^{IJ}\log(z-w)\,, \qquad \Psi^I(z)\Psi^J(w) \sim -\frac{\eta^{IJ}}{z-w}\,. \tag{3.14}$$

In terms of these, one gets

$$V_o^{(-1,-1)} = e^{2ik\cdot x}\,. \tag{3.15}$$

Here we used the fact that k has non-vanishing components only in the directions parallel to a D-brane, i.e. the N directions. Note that the matrix D is equal to $U = V$, satisfying the relations

$$DJ^R_\pm D = J^L_{\mp(\pm)}\,, \tag{3.16}$$

for the A-type (B-type) boundary conditions.

The picture-changing operators acting on the open string vertex operator are given by

$$G^\pm = \frac{i}{2}\Psi J^L_\pm \partial x\,. \tag{3.17}$$

Thus, the open string vertex operators with various superconformal ghost numbers are identical to the closed string vertex operators in the left sector with $k \to 2k$:

$$\begin{aligned} V_o^{(-1,0)} &= \left(kJ_+^L\Psi\right)e^{2ik\cdot x}\,, \\ V_o^{(0,-1)} &= \left(kJ_-^L\Psi\right)e^{2ik\cdot x}\,, \\ V_o^{(0,0)} &= 2\Big(-k\mathcal{J}^L\partial x + \left(kJ_+^L\Psi\right)\left(kJ_-^L\Psi\right)\Big)e^{2ik\cdot x}\,. \end{aligned} \tag{3.18}$$

Note that both α- and β-strings have the same form of the open string vertex operators except for the momentum. This fact will be important when we compute the scattering amplitudes of open strings on D-branes.

In the following we compute string scattering amplitudes in the presence of D-branes. The analysis is similar to that of scatterings in the presence of D-branes in $N = 1$ superstrings (For a review, see [17]).

3.2 The cylinder amplitude

Let us compute the one-loop amplitude of an $N = 2$ open string that connects two flat parallel Dp-branes (p refers to the number of both space- and time-like directions). This computation contains the information about the force between the D-branes [18]. As expected, we will obtain a non-vanishing force that is mediated by the closed string scalar ϕ. The D-branes of $N = 2$ strings do not posses RR charge and are not BPS objects.

The amplitude is defined by

$$\mathcal{A} = \int_0^\infty \frac{dt}{2t} \int du\, d\bar{u} \mathrm{tr}_\phi \left(e^{-2\pi t L_0} e^{2\pi i \theta J_0} \right) . \tag{3.19}$$

t is the modulus of a cylinder that corresponds to its radius. $u = \phi + it\theta$ denotes the $N = 2$ U(1) moduli. L_0, J_0 are the zero modes of the Virasoro generators and the $N = 2$ U(1) current. tr_ϕ is a summation over worldsheet fermionic modes $\psi_n, n \in Z + \phi$. L_0 takes the form

$$L_0 = \alpha' p_{||}^2 + \frac{r^2}{4\pi^2 \alpha'} + N\,, \tag{3.20}$$

where $p_{||}$ is the momentum along the D-branes worldvolume, r is the distance between the D-branes and N is the oscillation number. As in the closed string case [4], all the massive excitations do not contribute to the amplitude due to a cancellation between the matter and ghosts sectors.

We obtain

$$\begin{aligned}\mathcal{A} &= V_p \int_0^\infty dt\, \left(8\pi^2 \alpha' t\right)^{-p/2} e^{-\frac{tr^2}{2\pi\alpha'}} \\ &= 2\pi V_p \left(4\pi^2\alpha'\right)^{1-p} G_{4-p}(r)\,. \end{aligned}\tag{3.21}$$

V_p is the volume of the Dp-branes worldvolume and G_{4-p} is the harmonic function in a flat $\mathbb{R}^{4-p}$

$$G_{4-p}(r) = \frac{1}{4}\pi^{\frac{p-4}{2}}\, \Gamma\left(\frac{2-p}{2}\right) r^{p-2}\,. \tag{3.22}$$

The effective action of β-string is the Plebanski action [4] and that of α-string is a free real scalar action [5]. By adding an interaction term to Dp-branes we have

$$S = \frac{1}{2\kappa^2} \int d^4x\, (\partial\phi)^2 + \mu_p \int d^px\, \phi\,, \tag{3.23}$$

where we omitted the cubic interaction of the β-string effective action. Note that ϕ has dimension of length squared, and the target space metric obtained from ϕ is dimensionless. Consequently κ^2 has a length dimension 6, and μ_p has a mass dimension $p + 2$. The action implies that the amplitude $\mathcal{A}$ is

$$\mathcal{A} = \mu_p^2 \kappa^2 V_p\, G_{4-p}(r)\,. \tag{3.24}$$

Comparing (3.21) and (3.24) we get

$$\mu_p^2 = \frac{2\pi(4\pi^2\alpha')^{1-p}}{\kappa^2}\,. \tag{3.25}$$

Since the D-branes of $N = 2$ strings are not BPS objects there is a priori no reason to expect that this result is not corrected by higher order terms. However, this may still be the case since most of the higher order amplitudes of $N = 2$ strings vanish.

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3.3 Closed strings scattering off D-branes

Consider the scattering amplitudes of closed strings off a D-brane. The computation involves the insertion of closed string vertex operators on a disk with the boundary conditions

$$X_L^I(z) = D^I_J X_R^J(\bar z)\,, \qquad \psi_L^I(z) = D^I_J \psi_R^J(\bar z)\,, \tag{3.26}$$

at $z = \bar z$. We use the doubling technique to implement the boundary conditions for the matter as well as the ghost sectors.

We will use in the sequel the following formula for the ghost and bosonized superconformal ghosts correlators

$$\begin{aligned} \langle c(z_1)c(z_2)c(z_3)\rangle &= (z_1 - z_2)(z_2 - z_3)(z_1 - z_3), \\ \left\langle e^{-\varphi^\pm(z_1)} e^{-\varphi^\pm(z_2)}\right\rangle &= \frac{1}{z_1 - z_2}\,. \end{aligned} \tag{3.27}$$

3.3.1 One closed string

Here we calculate directly the coupling of a D-brane to the closed string scalar. It is given by the one-point function on the disk of the closed string vertex operator

$$A_c = \frac{1}{2\pi}\langle V_L^{(-1,-1)}(z) V_R^{(-1,-1)}(\bar z)\rangle_{D_2} \langle c(z)c(\bar z)\rangle_{D_2} \langle e^{-\varphi^+(z)} e^{-\varphi^+(\bar z)}\rangle_{D_2} \langle e^{-\varphi^-(z)} e^{-\varphi^-(\bar z)}\rangle_{D_2}\,. \tag{3.28}$$

The $1/2\pi$ factor comes from the division by the volume of the residual conformal Killing subgroup $\mathrm{U}(1) \subset \mathrm{SL}(2,\mathbb{R})$. Using the on-shell conditions $k + Dk = 0$, $k^2 = Dk^2 = 0$, and fixing $z = i$ (which is invariant is under the U(1)) we get

$$\mathcal{A}_c = \frac{1}{2\pi}\frac{|z-\bar z|^{k\cdot Dk}}{(z - \bar z)} = \frac{1}{2\pi(z-\bar z)} = -\frac{i}{4\pi}\,. \tag{3.29}$$

The scattering amplitude of one closed string and one open string in the presence of a D-brane is

$$\begin{aligned} \mathcal{A}_{co} &= \langle V_L^{(-1,-1)}(k_1, z) V_R^{(-1,-1)}(Dk_1, \bar z) V_o^{(0,0)}(k_2, w)\rangle_{D_2} \times \\ &\quad \times \langle c(z)c(\bar z)c(w)\rangle_{D_2} \langle e^{-\varphi^+(z)} e^{-\varphi^+(\bar z)}\rangle_{D_2} \langle e^{-\varphi^-(z)} e^{-\varphi^-(\bar z)}\rangle_{D_2} \\ &= |z-\bar z|^{k_1\cdot Dk_1} |z-w|^{2k_1\cdot k_2} |\bar z - w|^{2Dk_1\cdot k_2} \times \\ &\quad \times 2i\left(\frac{k_2 \mathcal{J}^L k_1}{w - z} + \frac{k_2\mathcal{J}^L Dk_1}{w - \bar z}\right)\frac{(z-w)(\bar z - w)}{z - \bar z}\,, \end{aligned} \tag{3.30}$$

where we have omitted the fermionic part of the open string vertex operator, which does not contribute to the amplitude.

The on-shell conditions are $k_1 + Dk_1 + 2k_2 = 0$, $k_1^2 = (Dk_1)^2 = k_2^2 = 0$. Fixing $z = i$, $w = 0$ we get

$$\mathcal{A}_{co} = i\left(k_2\mathcal{J}^L(k_1 - Dk_1)\right) = -2i(k_1\mathcal{J}^L k_2)\,. \tag{3.31}$$

The on-shell conditions imply that $k_1 - Dk_1$ and k_2 can be both non-zero only for the $(1+1)$-brane. Thus, only for the $(1+1)$-brane $\mathcal{A}_{co} \neq 0$.

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Two $\mathcal{A}_{co}$ amplitudes can be contracted by an open string exchange, resulting in an open string channel of $\mathcal{A}_{cc}$, the amplitude of two closed string states on a disk. This amplitude will be computed in the next subsection. The above result implies that $\mathcal{A}_{cc}$ in the presence of $(1+1)$-branes develops an open string pole. We will see that this is indeed the case.

3.3.2 Two closed strings

The net suerconformal ghost number on a disk must be 2 [15]. Thus, we take the following matter sector vertex operators

$$\begin{aligned} V_L^{(-1,0)}(z;k_1)V_R^{(-1,0)}(\bar{z};k_1) &= \frac{1}{4}(k_1 J_+^L \Psi(z))(k_1 J_+^R D\Psi(\bar{z}))e^{ik_1\cdot x(z)}e^{ik_1\cdot Dx(\bar{z})}\,, \\ V_L^{(0,-1)}(w;k_2)V_R^{(0,-1)}(\bar{w};k_2) &= \frac{1}{4}(k_2 J_-^L \Psi(w))(k_2 J_-^R D\Psi(\bar{w}))e^{ik_2\cdot x(w)}e^{ik_2\cdot Dx(\bar{w})}\,. \end{aligned} \tag{3.32}$$

The two point function becomes

$$\frac{1}{8}\left(\frac{|z-\bar{z}||w-\bar{w}|}{|z-\bar{w}|^2}\right)^s\left(\frac{|z-w|^2}{|z-\bar{w}|^2}\right)^t\left(\frac{2A}{(z-\bar{z})(w-\bar{w})}-\frac{2B}{|z-w|^2}+\frac{2C}{|z-\bar{w}|^2}\right). \tag{3.33}$$

Here

$$s = k_1\cdot Dk_1 = \frac{1}{2}(k_1+Dk_1)^2\,, \qquad t = k_1\cdot k_2 = \frac{1}{2}(k_1+k_2)^2\,, \tag{3.34}$$

and

$$\begin{aligned} A &= (k_1 J_+^L \eta^{-1} D J_-^R k_1)(k_2 J_-^L \eta^{-1} D J_+^R k_2)\,, \\ B &= (k_1 J_+^L \eta^{-1} J_+^L k_2)(k_1 J_+^R D\eta^{-1} D J_+^R k_2)\,, \\ C &= (k_1 J_+^L \eta^{-1} D J_+^R k_2)(k_1 J_+^R D\eta^{-1} J_+^L k_2)\,. \end{aligned} \tag{3.35}$$

Setting $z=i, w=iy$ with $0<y<1$ and integrating over y, we obtain the amplitude:

$$\mathcal{A}_{cc} = -\frac{i}{2}\left(A\,\frac{\Gamma(s-1)\Gamma(t+1)}{\Gamma(s+t)} + B\,\frac{\Gamma(s)\Gamma(t)}{\Gamma(s+t)} - C\,\frac{\Gamma(s)\Gamma(t+1)}{\Gamma(s+t+1)}\right). \tag{3.36}$$

In the computation, it was convenient to use a new variable x defined by $y=(1-\sqrt{x})/(1+\sqrt{x})$.

Define

$$J_\pm = \eta \pm i\mathcal{J}, \quad \widetilde{J}_\pm = \eta \pm i\widetilde{\mathcal{J}}\,. \tag{3.37}$$

One can check that

$$J_\pm \eta^{-1} J_\pm = 2J_\pm\,, \qquad J_\pm \eta^{-1} J_\mp = 0\,. \tag{3.38}$$

Define also

$$p_1 = k_1\,, \qquad p_2 = Dk_1\,, \qquad p_3 = Dk_2\,, \qquad p_4 = k_2\,, \tag{3.39}$$

which obey the relations

$$p_1+p_2+p_3+p_4 = 0\,, \qquad p_i^2 = 0\,. \tag{3.40}$$

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In terms of these,

$$s = p_1 \cdot p_2\,, \qquad t = p_2 \cdot p_3\,. \tag{3.41}$$

We also use

$$c_{ij} = -ip_i^I \mathcal{J}_{IJ} p_j^J\,. \tag{3.42}$$

c_{ij} obey the identities (2.7). In terms of the momenta p, the coefficients A, B, C can be written as

$$\begin{aligned} A &= \left(p_1 J_+^L \eta^{-1} D J_-^R D p_2\right)\left(p_4 J_-^L \eta^{-1} D J_+^R D p_3\right), \\ B &= 2\left(p_1 J_+^L p_4\right)\left(p_2 D J_+^R D \eta^{-1} D J_+^R D p_3\right), \\ C &= \left(p_1 J_+^L \eta^{-1} D J_+^R D p_3\right)\left(p_2 D J_+^R D \eta^{-1} J_+^L p_4\right)\,. \end{aligned} \tag{3.43}$$

Using (3.16), they take the form

- **A-type**

$$\begin{aligned} A &= 4\left(p_1 J_+^L p_2\right)\left(p_4 J_-^L p_3\right), \\ B &= 4\left(p_1 J_+^L p_4\right)\left(p_2 J_-^L p_3\right), \\ C &= 0\,. \end{aligned} \tag{3.44}$$

- **B-type**

$$\begin{aligned} A &= 0\,, \\ B &= 4\left(p_1 J_+^L p_4\right)\left(p_2 J_+^L p_3\right), \\ C &= 4\left(p_1 J_+^L p_3\right)\left(p_2 J_+^L p_4\right). \end{aligned} \tag{3.45}$$

They can be further simplified using the identities (2.7)

- **A-type**

$$\mathcal{A}_{cc} = 2i\,\frac{\Gamma(s-1)\Gamma(t)}{\Gamma(s+t)}(t-c_{14})(t+c_{23})\,. \tag{3.46}$$

- **B-type**

$$\mathcal{A}_{cc} = 0\,. \tag{3.47}$$

Recall that the B-type boundary conditions correspond to (2+2)-, (2+0)-, (0+2)- and (0+0)-branes in the β-string, and (2+1)- and (0+1)-branes in the α-string. The scattering amplitudes in these cases vanish suggesting a decoupling from the bulk. The A-type boundary conditions correspond to (1+1)-branes in the β-string and (1+2)- and (1+0)-branes in the α-string. The amplitudes in these cases contain infinite number of massive poles of both open string(s-) and closed string(t-) channels. This suggests that these D-branes do not provide consistent $N = 2$ boundary conditions.

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3.4 One closed and two open strings

We insert one closed string vertex operator and two open string vertex operators on a disk with the appropriate boundary conditions. The matter sector of the amplitude reads

$$\langle V_o^{(-1,0)}(z_1;k_1)V_o^{(-1,0)}(z_2;k_2)V_L^{(0,-1)}(z_3;k_3)V_R^{(0,-1)}(\bar{z}_3;k_3)\rangle_{D_2}\,, \tag{3.48}$$

where z_1, z_2 are real. Using the doubling technique we get

$$\begin{aligned}\Big\langle \left(k_1 J_+^L \Psi(z_1)\right) e^{2ik_1\cdot x(z_1)} \left(k_2 J_+^L \Psi(z_2)\right) e^{2ik_2\cdot x(z_2)} \left(k_3 J_-^L \Psi(z_3)\right) e^{ik_3\cdot x(z_3)} \times \\ \times \left(k_3 J_-^R D\Psi(\bar{z}_3)\right) e^{ik_1\cdot Dx(\bar{z}_3)}\Big\rangle\,.\end{aligned} \tag{3.49}$$

Using the momentum conservation law

$$2k_1 + 2k_2 + k_3 + Dk_3 = 0\,, \tag{3.50}$$

this becomes

$$\begin{aligned}\left(\frac{|z_1-z_3|^2|z_2-z_3|^2}{|z_1-z_2|^2|z_3-\bar{z}_3|^2}\right)^t \Bigg(\frac{-2}{(z_1-z_3)(z_2-\bar{z}_3)}\left(k_1 J_+^L k_3\right)\left(k_2 J_+^L \eta^{-1} D J_+^R k_3\right) + \\ + \frac{2}{(z_1-\bar{z}_3)(z_2-z_3)}\left(k_1 J_+^L \eta^{-1} D J_+^R k_3\right)\left(k_2 J_+^L k_3\right)\Bigg)\,.\end{aligned} \tag{3.51}$$

Here we used $t = 2k_1\cdot k_3$. The ghost sector part reads

$$\langle c(z_1)c(z_3)\tilde{c}(\bar{z}_3)\rangle_{D_2}\langle e^{-\varphi^+(z_1)}e^{-\varphi^+(z_2)}\rangle_{D_2}\langle e^{-\varphi^-(z_3)}e^{-\tilde{\varphi}^-(\bar{z}_3)}\rangle_{D_2} = \frac{|z_1-z_3|^2}{z_1-z_2}\,. \tag{3.52}$$

Setting $z_1 = 0$, $z_2 = x$ with $-\infty < x < \infty$ and $z_3 = i$ and integrating over x, we obtain the amplitude

$$\mathcal{A}_{coo} = 2i\pi\,\frac{\Gamma(1-2t)}{\Gamma(1-t)^2}\left[\left(k_1 J_+^L k_3\right)\left(k_2 J_+^L \eta^{-1} D J_+^R k_3\right) + \left(k_1 J_+^L \eta^{-1} D J_+^R k_3\right)\left(k_2 J_+^L k_3\right)\right]\,, \tag{3.53}$$

and we used $2^{1-2t}B(\frac{1}{2}-t,\frac{1}{2}) = 2\pi\Gamma(1-2t)/\Gamma(1-t)^2$. Define

$$p_1 = 2k_1\,,\qquad p_2 = 2k_2\,,\qquad p_3 = Dk_3\,,\qquad p_4 = k_3\,, \tag{3.54}$$

which obey

$$p_1 + p_2 + p_3 + p_4 = 0\,,\qquad p_i^2 = 0\,, \tag{3.55}$$

then

$$t = p_1\cdot p_4 = p_2\cdot p_3\,,\qquad s = p_1\cdot p_2 = p_3\cdot p_4 = -2t\,,\qquad u = p_1\cdot p_3 = p_2\cdot p_4 = -s-t = t\,. \tag{3.56}$$

In terms of these, the amplitude takes the form

$$\mathcal{A}_{ooc} = -i\pi t\,\frac{\Gamma(-2t)}{\Gamma(1-t)^2}\left[\left(p_1 J_+^L p_4\right)\left(p_2 J_+^L \eta^{-1} D J_+^R D p_3\right) + \left(p_1 J_+^L \eta^{-1} D J_+^R D p_3\right)\left(p_2 J_+^L p_4\right)\right]\,. \tag{3.57}$$

For the A-type boundary conditions, (3.16) and the relation $J_+\eta^{-1}J_- = 0$ imply that

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$\mathcal{A}_{ooc} = 0$. For the B-type boundary conditions, (3.16) and (2.7) imply that $\mathcal{A}_{ooc} = 0$. The space-time filling $(2+2)$-brane in the β-string is exceptional. In this case $D = 1$ so that $p_3 = p_4$. It thus follows that $s = t = u = 0$. Taking the limit $t \to 0$ in the amplitude, one obtains

$$\mathcal{A}_{ooc} = i\pi c_{12}^2 \,, \tag{3.58}$$

in agreement with the result of [16].

3.5 One open and two closed strings

We insert two closed string vertex operators and one open string vertex operator on a disk with the appropriate boundary conditions. The matter sector of the amplitude reads

$$\langle V_L^{(-1,0)}(z;k_1) V_R^{(-1,0)}(\bar{z};k_1) V_L^{(0,-1)}(w;k_2) V_R^{(0,-1)}(\bar{w};k_2) V_o^{(0,0)}(x;k_3)\rangle_{D_2} \,, \tag{3.59}$$

where x is real. Using the doubling technique we get

$$\begin{aligned}
&\frac{1}{8}\Big\langle \left(k_1 J_+^L \Psi(z)\right) e^{ik_1\cdot x(z)} \left(k_1 J_+^R D\Psi(\bar{z})\right) e^{ik_1\cdot Dx(\bar{z})} \left(k_2 J_-^L \Psi(w)\right) e^{ik_2\cdot x(w)} \cdot \\
&\quad \cdot \left(k_2 J_-^R D\Psi(\bar{w})\right) e^{ik_2\cdot Dx(\bar{w})} \left(-k_3 \mathcal{J}^L \partial x(x) + (k_3 J_+^L\Psi(x))(k_3 J_-^L \Psi(x))\right) e^{2ik_3\cdot x(x)} \Big\rangle .
\end{aligned} \tag{3.60}$$

It is easy to show that this takes the form

$$\begin{aligned}
&\frac{1}{8}\left(|z-\bar{z}|^{k_1\cdot Dk_1}|w-\bar{w}|^{k_2\cdot Dk_2}|z-w|^{2k_1\cdot k_2}|z-\bar{w}|^{2k_1\cdot Dk_2}|z-x|^{4k_1\cdot k_3}|w-x|^{4k_2\cdot k_3}\right) \times \\
&\times\Bigg[i\left(\frac{A}{(z-\bar{z})(w-\bar{w})} - \frac{B}{|z-w|^2} + \frac{C}{|z-\bar{w}|^2}\right) \times \\
&\quad \times\left(\frac{k_3\mathcal{J}^L k_1}{x-z} + \frac{k_3\mathcal{J}^L Dk_1}{x-\bar{z}} + \frac{k_3\mathcal{J}^L k_2}{x-w} + \frac{k_3\mathcal{J}^L Dk_2}{x-\bar{w}}\right) + \\
&\quad + \frac{E}{(z-x)(\bar{z}-x)(w-\bar{w})} + \frac{F}{(z-\bar{z})(w-x)(\bar{w}-x)} + \frac{G}{(z-x)(w-x)(\bar{z}-\bar{w})} + \\
&\quad + \frac{H}{(\bar{z}-x)(\bar{w}-x)(z-w)} + \frac{I}{(z-x)(\bar{w}-x)(\bar{z}-w)} + \frac{J}{(\bar{z}-x)(w-x)(z-\bar{w})}\Bigg],
\end{aligned} \tag{3.61}$$

where A, B, C are given in (3.35), and

$$\begin{aligned}
E &= -(k_1 J_+^L \eta^{-1} J_+^L k_3)(k_1 J_+^R D\eta^{-1} J_-^L k_3)(k_2 J_-^L \eta^{-1} D J_+^R k_2)\,, \\
F &= (k_1 J_+^L \eta^{-1} D J_-^R k_1)(k_2 J_-^L \eta^{-1} J_-^L k_3)(k_2 J_-^R D\eta^{-1} J_+^L k_3)\,, \\
G &= (k_1 J_+^L \eta^{-1} J_+^L k_3)(k_2 J_-^L \eta^{-1} J_-^L k_3)(k_1 J_+^R D\eta^{-1} D J_+^R k_2)\,, \\
H &= \Big((k_1 J_+^R D\eta^{-1} J_+^L k_3)(k_2 J_-^R D\eta^{-1} J_-^L k_3) - \\
&\qquad - (k_1 J_+^R D\eta^{-1} J_-^L k_3)(k_2 J_-^R D\eta^{-1} J_+^L k_3)\Big)(k_1 J_+^L \eta^{-1} J_+^L k_2), \\
I &= -(k_1 J_+^L \eta^{-1} J_+^L k_3)(k_2 J_-^R D\eta^{-1} J_-^L k_3)(k_1 J_+^R D\eta^{-1} J_+^L k_2), \\
J &= -(k_1 J_+^R D\eta^{-1} J_+^L k_3)(k_2 J_-^L \eta^{-1} J_-^L k_3)(k_1 J_+^L \eta^{-1} D J_+^R k_2)\,.
\end{aligned} \tag{3.62}$$

One can verify that for A-type D-branes $C = I = J = 0$, while for B-type D-branes $A = E = F = 0$.

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Setting $z = i$ and $w = iy$ with $0 < y < 1$ and integrating over x and y, we obtain

$$
\begin{aligned}
\mathcal{A}_{cco} = &-i2^{k_1\cdot Dk_1+k_2\cdot Dk_2-4} \times \\
&\times \int_0^1 dy \int_{-\infty}^{\infty} dx y^{k_2\cdot Dk_2-1}(1-y)^{2k_1\cdot k_2+1}(1+y)^{2k_1\cdot Dk_2+1}(x^2+1)^{2k_1\cdot k_3}(x^2+y^2)^{2k_2\cdot k_3} \times \\
&\times \Bigg[-\left(\frac{A}{-4y}+\frac{(1-y)^2C-(1+y)^2B}{(1-y^2)^2}\right)\cdot\left(\frac{k_3\mathcal{J}^L(k_1-Dk_1)}{x^2+1}+y\frac{k_3\mathcal{J}^L(k_2-Dk_2)}{x^2+y^2}\right)+ \\
&\quad + \frac{-i}{2}\frac{E}{(x^2+1)y}+\frac{-i}{2}\frac{F}{x^2+y^2}+i(G-H)\frac{x^2-y}{(x^2+1)(x^2+y^2)(1-y)}+ \\
&\quad + i(I-J)\frac{x^2+y}{(x^2+1)(x^2+y^2)(1+y)}\Bigg]. \qquad (3.63)
\end{aligned}
$$

For (2+2)-branes $A = E = F = 0$ and $k_i = Dk_i$. Thus, $G = H$ and $I = J$, implying that $\mathcal{A}_{cco} = 0$. Also, for (2+0)-, (0+2)-, (1+0)-, (0+1)- and (0+0)-branes the on-shell condition $k_3^2 = 0$ means $k_3 = 0$ and therefore $\mathcal{A}_{cco} = 0$. For (2+1)-branes, the amplitude should not have poles: it is found that the residues of the open and closed string massless poles take the form $\mathcal{A}_{co}\cdot\mathcal{A}_{coo}$ and $\mathcal{A}_{co}\cdot\mathcal{A}_{ccc}$ respectively, which were shown to vanish. In order to see explicitly if unphysical massive poles disappear in the amplitude, we have to perform the integration. We verified that $\mathcal{A}_{cco} = 0$ at least for the case $k_1\cdot k_3 = 0$. We will leave it as an open problem to show this for a generic value of $k_1\cdot k_3$.

3.6 Three and four-point amplitudes of open strings

We insert three (four) open string vertex operators on a disk with appropriate boundary conditions at the boundary. Open string vertex operators on any type of D-branes are obtained by setting to zero the components of the momentum transverse to the D-branes $k_\perp$. This means that the scattering amplitudes are given by setting $k_\perp$ to zero in the amplitude of the space-time filling (2+2)-branes [16]:

$$\mathcal{A}_{ooo} = c_{12}\,, \qquad \mathcal{A}_{oooo} = 0\,. \qquad (3.64)$$

The effective action on the worldvolume of any D-brane is obtained by a dimensional reduction of the effective action of the $(2+2)$-brane. Using [16], we see that D-branes effective actions in $\mathcal{N} = 2$ strings are given by a dimensional reduction of self-dual Yang-Mills theory.

Note that apart from the space-time filling (2+2)-branes, only the B-type (2+1)-branes in α-string allow non-trivial propagating degrees of freedom on the worldvolume, and the above three-point amplitude may be non-trivial. However, it turns out to vanish for kinematical reasons. To see this, we notice that an on-shell momentum on the (2+1)-brane takes (in the real basis) the form

$$k_i^I = (k_i\cos\theta_i,\ k_i\sin\theta_i,\ k_i,\ 0)\,. \qquad (3.65)$$

In terms of this parametrization, $\mathcal{A}_{ooo}$ reads

$$c_{ij} = -ik_ik_j\sin(\theta_i-\theta_j)\,. \qquad (3.66)$$

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Using the momentum conservation law, we find

$$\cos(\theta_i - \theta_j) = 1\,, \tag{3.67}$$

and $\mathcal{A}_{ooo} = 0$ for the $(2+1)$-brane. Thus, we find that lower-dimensional D-branes in $N = 2$ strings allow no non-trivial open string scattering amplitudes.

3.7 Summary

Let us summarize the above computations. We computed several disk amplitudes for strings in the presence of A-type and B-type D-branes. We found that the one-point function of closed strings is $\mathcal{A}_c = -i/4\pi$ for all the D-branes. The two-point function $\mathcal{A}_{cc}$ vanishes for the B-type D-branes but exhibits infinite number of massive poles for A-type D-branes, suggesting an inconsistency.

We found that for all D-branes except the $(1+1)$-brane and the space-time filling $(2+2)$-brane, the open-closed amplitudes, $\mathcal{A}_{co}$ and $\mathcal{A}_{ooc}$, vanish. The disk amplitudes with two or three closed strings and at least one open string vertex operators vanish for the $(0+0)$-, $(2+0)$-, $(0+2)$-, $(1+0)$ and $(0+1)$-branes. This is due to the fact that the inserted open string vertex operators can be chosen to be in the (0,0)-picture, which vanish kinematically by imposing the on-shell condition. Thus, for these D-branes all the open-closed amplitudes on a disk vanish. It is likely that for these cases all higher genus amplitudes vanish, since picture-changing operators acting on open string vertex operators always yield a vanishing operator due to the kinematics.

For the (2+2)-, (2+1)-, (1+2)-, (1+1)-branes, the open-closed amplitudes could be non-zero. In fact, we found that for the $(1+1)$-brane $\mathcal{A}_{co} = -2i(k_1\mathcal{J}^L k_2)$, and for the (2+2)-brane $\mathcal{A}_{coo} = i\pi c_{12}^2$. However, $\mathcal{A}_{cco} = 0$ for the (2+2)-brane, giving no unphysical open and closed string poles.

We found that for all the D-branes except the (2+2)-brane, $\mathcal{A}_{ooo}$ and $\mathcal{A}_{oooo}$ vanish. The amplitudes with more than four open string vertices and no closed string vertex operator should vanish too, as these are given by a dimensional reduction of the corresponding amplitudes for the $(2+2)$-brane which vanish [19]. As discussed in [19], the vanishing of n-point open string amplitudes is valid for any order of string loops. Only the space-time filling $(2+2)$-brane has a non-trivial amplitude $\mathcal{A}_{ooo} = c_{12}$ [16].

4. The effective D-branes worldvolume theory

As discussed in the previous section, the effective action on D-branes is given by dimensional reduction of self-dual Yang-Mills(SDYM) theory in (2+2)-dimensions. The self-dual equation reads

$$F_{IJ} = \frac{1}{2}\epsilon_{IJKL}F^{KL}\,, \tag{4.1}$$

where

$$F_{IJ} = -[D_I, D_J]\,, \qquad D_I = \partial_I - A_I\,. \tag{4.2}$$

In components,

$$[D_1, D_2] = [D_3, D_4]\,, \qquad [D_2, D_3] = -[D_1, D_4]\,, \qquad [D_3, D_1] = -[D_2, D_4]\,. \tag{4.3}$$

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$(2+2) \to (2+1)$. Define $A_\mu = (A_1, A_2, A_3)$, $A_4 = \phi$ and regard all the fields as x_4-independent. The self-dual equations become

$$D^\mu \phi = -\frac{1}{2}\epsilon^{\mu\nu\rho}[D_\nu, D_\rho]\,. \tag{4.4}$$

This is the "Bogomolny equation" on $\mathbb{R}^{2,1}$.

$(2+2) \to (2+0)$. Define $A_3 = \phi_3$, $A_4 = \phi_4$ and regard all the fields as x_3, x_4-independent. One gets

$$[D_1, D_2] = [\phi_3, \phi_4]\,, \qquad D_2\phi_3 = -D_1\phi_4\,, \qquad D_1\phi_3 = D_2\phi_4\,. \tag{4.5}$$

In terms of the complex scalar defined by $\Phi = \phi_3 - i\phi_4$, the above equations become

$$[D_1, D_2] = -\frac{i}{2}[\Phi, \Phi^\dagger]\,, \qquad (D_1 - iD_2)\Phi = 0\,. \tag{4.6}$$

This is the "Hitchin" system. Recall that if we start from SDYM in $\mathbb{R}^4$ of signature (4,0), we end up instead of (4.6) with

$$[D_1, D_2] = -\frac{i}{2}[\Phi, \Phi^\dagger]\,, \qquad (D_1 + iD_2)\Phi = 0\,. \tag{4.7}$$

$(2+2) \to (0+1)$. Define $A_1 = \phi_1 = -\phi^1$, $A_2 = \phi_2 = -\phi^2$, $A_3 = \phi_3 = +\phi^3$ and regard all the fields as x_1, x_2, x_3-independent. One obtains

$$D_4\phi^i = \frac{1}{2}\epsilon^{ijk}[\phi_j, \phi_k]\,. \tag{4.8}$$

This is the "Nahm equation". It is known that the solutions of the Nahm equation are related by Nahm transformation to those of the Bolomolny equation. Nahm transformation is a T-duality that relates (2+1)-branes with (0+1)-branes.

$(2+2) \to (0+0)$. Define $A_I = \phi_I = \pm\phi^I$ (depending on the signature) and regard all the fields as x^I independent. The self-dual equations become

$$[\phi_I, \phi_J] = \frac{1}{2}\,\epsilon_{IJKL}[\phi^K, \phi^L]\,. \tag{4.9}$$

This is the "ADHM" equation.

5. D-brane gravitational backgrounds

In this section we construct $N=2$ D-brane gravitational backgrounds. The effective action of the β-string with a source term coupling to D-branes is

$$S_\beta = \int d^4x \left(\eta^{i\bar{j}}\,\partial_i\phi\,\partial_{\bar{j}}\phi + \frac{1}{3}\,\phi\,\partial_j\partial_{\bar{i}}\phi\,\epsilon^{ij}\epsilon^{\bar{i}\bar{j}}\,\partial_i\partial_{\bar{j}}\phi\right) + \kappa\mu_p\int d^4x\,\phi\,\delta^{4-p}(x_\perp)\,, \tag{5.1}$$

where $x_\perp$ are the Dirichlet directions. The field equations read

$$-\partial^2\phi + \partial_1\partial_{\bar{2}}\phi\,\partial_2\partial_{\bar{1}}\phi - \partial_1\partial_{\bar{1}}\phi\,\partial_2\partial_{\bar{2}}\phi + \frac{\kappa\mu_p}{2}\,\delta^{4-p}(x_\perp) = 0\,. \tag{5.2}$$

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The first three terms correspond to the Plebanski equation [4], which is the Ricci flatness condition of the target space Kähler manifold. The scalar field is identified with the deformation of the flat space Kähler potential.

The effective action of the α-string with a D-brane source is [5]

$$S_\alpha = \int d^4x\, \eta^{i\bar{j}}\, \partial_i\phi\, \partial_{\bar{j}}\phi + \kappa\mu_p \int d^4x\, \phi\, \delta^{4-p}(x_\perp)\,. \tag{5.3}$$

The scalar field corresponds to a deformation of a potential $\widetilde{K}$ around a flat background.

$$\widetilde{K} = \eta_{1\bar{1}} z^1 \bar{z}^{\bar{1}} - \eta_{2\bar{2}} z^2 \bar{z}^{\bar{2}} + \kappa\phi\,. \tag{5.4}$$

The target space geometry is encoded $\widetilde{K}$ [14, 6]

$$\begin{aligned} g_{1\bar{1}} &= \partial_1\partial_{\bar{1}}\widetilde{K}\,, & g_{2\bar{2}} &= -\partial_2\partial_{\bar{2}}\widetilde{K}\,, \\ B_{1\bar{2}} &= \partial_1\partial_{\bar{2}}\widetilde{K}\,, & B_{\bar{1}2} &= \partial_{\bar{1}}\partial_2\widetilde{K}\,, \\ e^{2\Phi} &= g_{1\bar{1}}\,, \end{aligned} \tag{5.5}$$

where Φ is the dilaton. Note that in the $N = 2$ σ-model description z^1 corresponds to an $N = 2$ chiral superfield while z^2 corresponds to a twisted $N = 2$ chiral superfield.

In order to construct the D-branes gravitational backgrounds we assume translational invariance along the worldvolume directions and rotational invariance along the transverse directions.

We first consider D-branes in β-string.

(2+2)-brane. The field equation is

$$-\partial^2\phi + \partial_1\partial_{\bar{2}}\phi\partial_2\partial_{\bar{1}}\phi - \partial_1\partial_{\bar{1}}\phi\partial_2\partial_{\bar{2}}\phi + \frac{\kappa\mu_p}{2} = 0\,. \tag{5.6}$$

The space-time filling brane has no transverse directions. If we impose the translational invariance along the world volume directions, ϕ must be a constant. However, no constant solutions are allowed.

(2+0)-branes. Assuming the translational invariance along the worldvolume directions z_1 and the rotational invariance along the transverse directions, ϕ must take the form

$$\phi = \phi(|z_2|)\,. \tag{5.7}$$

The field equation reads

$$-\partial_2\partial_{\bar{2}}\phi + \frac{\kappa\mu_p}{2}\,\delta^2(z_2) = 0\,. \tag{5.8}$$

Thus, ϕ is given by the harmonic function in $\mathbb{R}^2$:

$$\phi = \frac{\kappa\mu_2}{4\pi}\,\log|z_2|^2\,. \tag{5.9}$$

The Kähler potential is given by $K = -|z_1|^2 + |z_2|^2 + \kappa\phi$ and the target space metric is

$$g_{1\bar{1}} = -1\,, \qquad g_{2\bar{2}} = 1 + \frac{\kappa^2\mu_2}{2}\delta^2(z_2)\,. \tag{5.10}$$

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The Riemann tensor reads

$$R_{i\bar{j}} = \partial_i \partial_{\bar{j}} \log \det g_{i\bar{j}} = \partial_i \partial_{\bar{j}} \log \left(1 + \frac{\kappa^2 \mu_2}{2} \delta^2(z_2) \right) . \tag{5.11}$$

The geometry is singular at the location of the branes, $z_2 = 0$. Note that

$$\phi = \frac{\kappa \mu_2}{4\pi} \left(\log z_2 + \log \bar{z}_2 \right) , \tag{5.12}$$

suggesting that ϕ is a singular Kähler transformation.

(0+0)-brane. Consider the (4,0) signature. We take

$$\phi = \phi(r) , \tag{5.13}$$

with $r^2 = |z_1|^2 + |z_2|^2$. The interaction terms can not be neglected and ϕ is not a harmonic function in $\mathbb{R}^4$. We will construct a smeared solution later.

Consider next the D-branes of the α-string. ϕ is given by the harmonics function in $\mathbb{R}^{4-p}$.

(2+1)-branes. We find

$$\phi = \frac{\kappa \mu_3}{2} |x_4| . \tag{5.14}$$

Using the formula (5.5), the target space metric and B-fields are given by

$$\begin{aligned} g_{1\bar{1}} &= -1 , \qquad g_{2\bar{2}} = 1 + \frac{\kappa^2 \mu_3}{2} \delta(x_4) , \\ B_{1\bar{2}} &= B_{\bar{1}2} = 0 . \end{aligned} \tag{5.15}$$

(0+1)-branes. As before, we do the analytic continuation. Then

$$\begin{aligned} g_{1\bar{1}} &= 1 + \frac{\kappa^2 \mu_1}{8\pi} \frac{2x_3^2 - x_1^2 - x_2^2}{r^5} , \qquad & g_{2\bar{2}} &= 1 + \frac{\kappa^2 \mu_1}{8\pi} \frac{2x_3^2 - x_1^2 - x_2^2}{r^5} - \frac{\kappa^2 \mu_1}{2} \delta^3(x_\perp) , \\ B_{1\bar{2}} &= -\frac{3\kappa^2 \mu_1}{8\pi} \frac{(x_1 - i x_2) x_3}{r^5} , \qquad & B_{\bar{1}2} &= -\frac{3\kappa^2 \mu_1}{8\pi} \frac{(x_1 + i x_2) x_3}{r^5} . \end{aligned} \tag{5.16}$$

Using this background we can construct a (0+0)-branes background by a Legendre transformation [3]. This yields a smeared solution rather than a fully localized one. The potential of the (0+1)-branes background is given by

$$\begin{aligned} \widetilde{K} &= |z_1|^2 - |z_2|^2 + \phi \\ &\sim |z_1|^2 - \frac{1}{2}(z_2 + \bar{z}_2)^2 - \frac{\kappa \mu_1}{4\pi} \frac{1}{\sqrt{2|z_1|^2 + \frac{1}{2}(z_2 + \bar{z}_2)^2}} . \end{aligned} \tag{5.17}$$

Here we used a symmetry transformation that leaves the background invariant. The Kähler potential of β-string is given by the Legendre transformation:

$$K(z_1, \bar{z}_1, w_2 + \bar{w}_2) \equiv \widetilde{K} - (w_2 + \bar{w}_2)(z_2 + \bar{z}_2) , \tag{5.18}$$

where $z_2 + \bar{z}_2$ can be written as a function of $w_2 + \bar{w}_2$ by solving

$$w_2 + \bar{w}_2 = \frac{\partial \widetilde{K}}{\partial (z_2 + \bar{z}_2)} = -(z_2 + \bar{z}_2) + \frac{\kappa \mu_1}{8\pi} \frac{z_2 + \bar{z}_2}{\left(2|z_1|^2 + \frac{1}{2}(z_2 + \bar{z}_2)^2\right)^{3/2}} . \tag{5.19}$$

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6. Discussion

In this section we would like to discuss some aspects of possible open/closed $N = 2$ string dualities. A natural approach would be to use the strategy employed for $N = 1$ superstrings. There, one considers two descriptions of D-branes: the perturbative description of D-branes as hyperplanes on which open strings end, and the supergravity description as solitonic solutions of the field equations. One then uses the decoupling limit in order to separate the open string degrees of freedom from the closed string ones [20]. The decoupling limit for D-branes of $N = 1$ superstrings is the low energy limit $l_s \to 0$. One may expect that in our case such a limit is already implemented since the $N = 2$ strings have no massive string states. Indeed, one support for this seems to be the worldsheet computation of open and closed strings correlators in section 3. The vanishing of the correlators without taking any decoupling limit indicates that at least perturbatively, open and closed strings decouple.

However, upon taking the decoupling limit, the supergravity backgrounds of D-branes in $N = 1$ superstrings provide a solution of the low-energy $N = 1$ closed string equations. Here we have solutions of the closed string field equations modified by a source term. Thus, without taking a limit we do not have an $N = 2$ closed string background.

We may try to remove the source term by taking another limit. In this case we are back in a flat background without a backreaction of the D-branes. It is possible that indeed in flat space the D-branes backreaction is trivial. It means that we should think about the $N = 2$ string as a topological string with really non-trivial structure of amplitudes only in curved spaces.

In curved spaces one could imagine that the duality would work as in topological field theories [21] or the, recently discussed, $c = 1$ non-critical strings [22]. One would expect that the effect of D-branes on the curved background would be to modify some moduli of the $N = 2$ closed string background. Let us see how this can happen.

Consider, for instance, the $(2 + 0)$ D-brane. As shown in section 4, the effective worldvolume theory is the Hitchin system. Let the curved space be of the form $T^*\Sigma_g$, where Σ is a genus g Riemann surface. When the signature is $(4, 0)$ $g = 0, 1$ while when the signature is $(2, 2)$ $g \geq 1$. Such $(2, 2)$ signature solutions have been constructed in [4]. The Hitchin system in this case consists of a flat U(1) connection and an harmonic one-form on Σ_g. The harmonic one-form can be viewed as an infinitesimal deformation of Σ_g to a Riemann surface Σ'_g. One can view then the effect of the D-brane as a modification of the $T^*\Sigma_g$ background. The choice of a flat U(1) connection can be mapped to a deformation of $T^*\Sigma_g$ by tensoring the cotangent bundle by it. Thus, one may conjecture that $N = 2$ closed strings on $T^*\Sigma_g$ with a $(2 + 0)$ D-brane is equivalent to $N = 2$ closed strings on the deformed $T^*\Sigma_g$.

Another possible approach to finding a duality could be a flop transition. This idea is an extension of the G_2 flop [23], which was employed to explain the geometric transition [21]. Consider as before the (2+0)-branes of the β-string on $T^*\Sigma_g$, with the D-branes wrapping the base Σ_g. The D-branes create a singularity of the background in their position. We can look for a smooth background by going smoothly to a negative volume of Σ_g. Thus,

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another possible duality is between $N = 2$ closed strings in the presence of D-branes and $N = 2$ closed strings on the background after the flop transition.

We expect that a similar discussion applies to the other D-branes. It seems interesting to explore such possible dualities by computing the topological amplitudes. The techniques of [24] could be useful.

Acknowledgments

We would like to thank Z. Yin for participation in the early stages of this work, and O. Aharony and D. Kutasov for a valuable discussion. This research is supported by the US-Israel Binational Science Foundation.

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